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Robust Mean Field Games of Autonomous Traffic via Physics-Informed Adversarial Attention Networks

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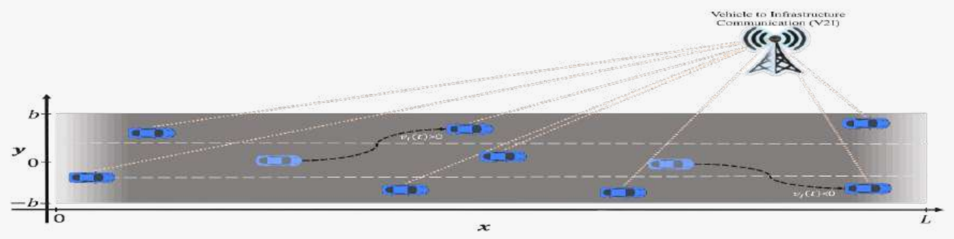
Introduction

Rapid technological advancements have led to a surge in AVs sharing roads with human-driven vehicles, creating complex traffic dynamics. Understanding these interactions is crucial for ensuring safety and optimising traffic efficiency. As traffic flow represents a complex non-equilibrium system of interacting entities, accurate modelling is essential to capture and predict its behaviour under varying conditions. In this work, we model the flow of AVs on multi-lane roadways, incorporating the lane-changing manoeuvres via the 2-dimensional RMFG framework.



In this work, we further propose a physics-informed attention-based GAN framework with residual-driven adaptive sampling [1] to efficiently model the coupled forward Fokker–Planck–Kolmogorov (FPK) and backward Hamilton–Jacobi–Bellman–Isaacs (HJBI) equations in the mean-field traffic flow setting. This unified approach overcomes the numerical challenges of solving forward–backward systems by leveraging GANs' learning capability to ensure stability, accuracy, and computational efficiency.

Robust Mean-Field Game

Consider the movement of a large number of AVs on a unidirectional highway of length L and width $2b$ for the planning horizon $[0, T]$. Let $\rho(t,x,y)$, $u(t,x,y)$ and $v(t,x,y)$ be the continuum density distribution (ρ) and the mean-field longitudinal and lateral velocities of the traffic stream, respectively. Further assume all the AVs are subject to adversarial disturbance that can arise from various sources, hampering their movement on the road. In the RMFG framework, we assume the AVs compete with each other to move on the road while minimising a certain cost objective function, which can be to minimise the fuel consumption, risk exposure, etc. Mathematically, we can formulate the **robust optimal control problem** of a representative agent to a **robust mean-field game** of coupled forward-in-time **Fokker-Planck-Kolmogorov (FPK)** and backwards-in-time **Hamilton-Jacobi-Bellman-Isaacs (HJBI)** equations given by

$$\text{Robust MFG}\begin{cases} \textbf{FPK:} & \partial_t\rho + \partial_x(\rho(u+d_1)) + \partial_y(\rho(v+d_2)) = \partial_{yy}\rho \\ \textbf{HJBI:} & \partial_t C + f^*(\partial_x C, \partial_y C) - \kappa^2(d_1^2+d_2^2) + \frac{\sigma^2(t)}{2}\partial_{yy}C \\ & u(t,x,y) = \arg\min\{f(t,u,v,\rho) + (u+d_1)\partial_x C\} \\ & v(t,x,y) = \arg\min\{f(t,u,v,\rho) + (v+d_2)\partial_y C\} \\ & d_1(t,x,y) = \frac{1}{2\kappa^2}\partial_x C, d_2(t,x,y) = \frac{1}{2\kappa^2}\partial_y C, \end{cases} \tag{1}$$

Attention Neural Network

The proposed model utilises the **generative adversarial framework (GAN)** [2] to model the robust MFG. Our proposed GAN utilises two modified versions of physics-informed neural networks [3], consisting of transformer networks that encode the input variables to a higher-dimensional feature space. Each hidden layer is then updated with the pointwise multiplication operation as follows

$$\begin{aligned} &\textbf{Encoder 1:} && \mathbf{U} = \Psi_U(\mathbf{W}_U\mathbf{X} + \mathbf{b}_U) \\ &\textbf{Encoder 2:} && \mathbf{V} = \Psi_V(\mathbf{W}_V\mathbf{X} + \mathbf{b}_V) \\ &\textbf{Hidden Layer } l: && \mathbf{Z}^{[l]} = \mathbf{W}^{[l]}\mathbf{H}^{[l-1]} + \mathbf{b}^{[l]}, \quad l = 1,2,\cdots,L-1 \\ & && \mathbf{H}^{[l]} = \psi^{[l]}(\mathbf{Z}^{[l]}), \quad l = 1,2,\cdots,L-1 \\ & && \mathbf{H}^{[l]} = (\mathbf{1} - \mathbf{H}^{[l]}) \odot \mathbf{U} + \mathbf{H}^{[l]} \odot \mathbf{V} \\ &\textbf{Output Layer } L: && \mathbf{H}^{[L]} = \psi^{[L]}(\mathbf{W}^{[L]}\mathbf{H}^{[L-1]} + \mathbf{b}^{[L]}) \end{aligned}$$

where $H^{[0]} = X$ is the $N \times 3$ design matrix consisting of points for initial/terminal and/or boundary conditions along with the points for physics-based regularisation of the FPK and HJBI-nets, respectively.

Robust Mean-Field Games for Macroscopic Traffic Flow Modelling

The MFG offers a deeper generalisation compared to traditional macroscopic models by considering the vehicles as rational and utility-optimising agents aiming to minimise some form of **travel cost**. Based on a specification of cost functions, a diverse set of models can be obtained.

Equilibrium Robust MFG

For the cost function

$$f(t,u,v,\rho) = \frac{1}{2}(U_{eq}(\rho) - u)^2 + \frac{1}{2}(V_{eq}(\rho) - v)^2, \tag{2}$$

We obtain the HJBI equation of the Equilibrium Robust MFG (EMFG) as

$$\begin{aligned} &\partial_t C + \frac{1}{2\kappa^4}(2\kappa^2 - 1 - 2\kappa^4)(\partial_x C^2 + \partial_y C^2) + (U_{eq}(\rho) + d_1)\partial_x C \\ &+ (V_{eq}(\rho) + d_2)\partial_y C + \sigma^2(t)\,\partial_{yy}C = 0 \\ &u(t,x,y) = U_{eq}(\rho), \quad v(t,x,y) = V_{eq}(\rho) \\ &d_1(t,x,y) = 0, \quad d_2(t,x,y) = 0 \end{aligned} \tag{3}$$

Non-Equilibrium Robust MFG

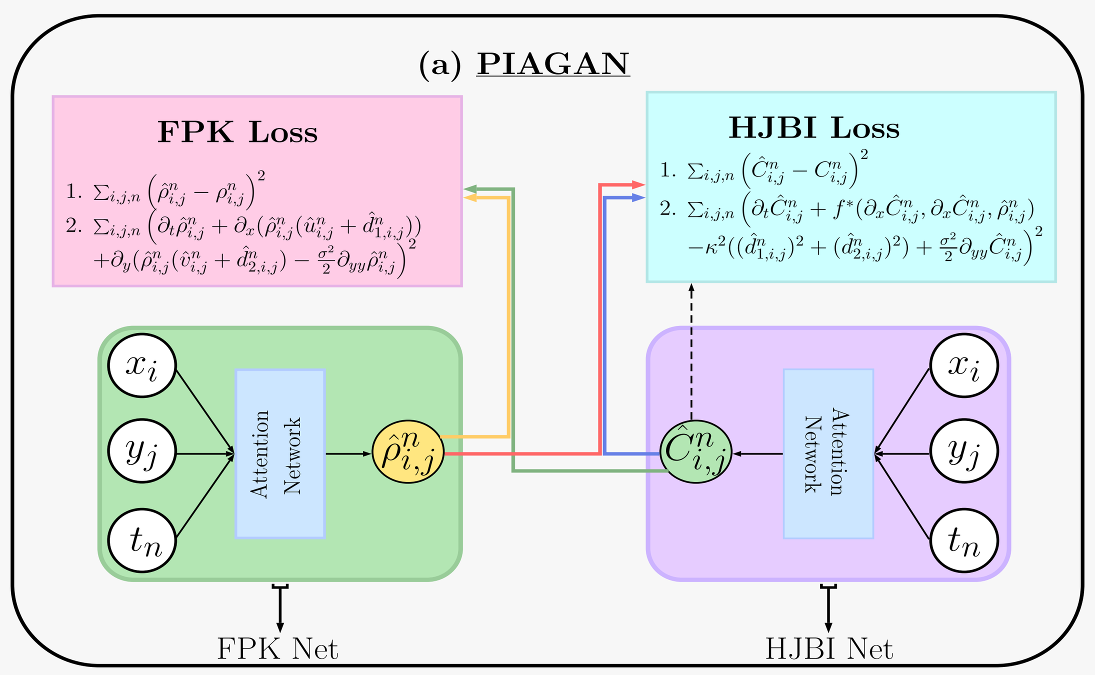
For the cost function

$$\mathbf{f}(t,\mathbf{u},\mathbf{v},\boldsymbol{\rho}) = \frac{1}{2}(\mathbf{U}_{eq}(\boldsymbol{\rho}) - \mathbf{u})^2 + \frac{1}{2}(\mathbf{V}_{eq}(\boldsymbol{\rho}) - \mathbf{v})^2 + \frac{\mathbf{v}^2\boldsymbol{\rho}}{\mathbf{v}_{max}\boldsymbol{\rho}_{max}} \tag{4}$$

where v_{max} is the free flow velocity in the lateral direction and ρ_{max} is the jam density, we obtain the HJBI of the non-equilibrium Robust MFG (NE-MFG) as

$$\begin{aligned} &\mathbf{C}_t + \frac{1}{2}(\mathbf{U}_{eq}(\boldsymbol{\rho}) - \mathbf{u})^2 + \frac{1}{2}(\mathbf{V}_{eq}(\boldsymbol{\rho}) - \mathbf{v})^2 + \frac{\mathbf{v}^2\boldsymbol{\rho}}{\mathbf{v}_{max}\boldsymbol{\rho}_{max}} - \boldsymbol{\kappa}^2(\mathbf{d}_1^2 + \mathbf{d}_2^2) \\ &+ (\mathbf{u} + \mathbf{d}_1)\partial_{\mathbf{x}}\mathbf{C} + (\mathbf{v} + \mathbf{d}_2)\partial_{\mathbf{y}}\mathbf{C} + \frac{\boldsymbol{\sigma}^2(t)}{2}\partial_{\mathbf{yy}}\mathbf{C} = 0, \\ &\mathbf{u}(t,\mathbf{x},\mathbf{y}) = \mathbf{U}_{eq}(\boldsymbol{\rho}) - \partial_{\mathbf{x}}\mathbf{C}, \quad \mathbf{v}(t,\mathbf{x},\mathbf{y}) = \frac{\mathbf{V}_{eq}(\boldsymbol{\rho}) - \partial_{\mathbf{y}}\mathbf{C}}{1 + \frac{\boldsymbol{\rho}}{\mathbf{v}_{max}\boldsymbol{\rho}_{max}}}, \\ &\mathbf{d}_1(t,\mathbf{x},\mathbf{y}) = \frac{1}{2\boldsymbol{\kappa}^2}\partial_{\mathbf{x}}\mathbf{C}, \quad \mathbf{d}_2(t,\mathbf{x},\mathbf{y}) = \frac{1}{2\boldsymbol{\kappa}^2}\partial_{\mathbf{y}}\mathbf{C}. \end{aligned} \tag{5}$$

Physics-Informed Adversarial Network of MFG

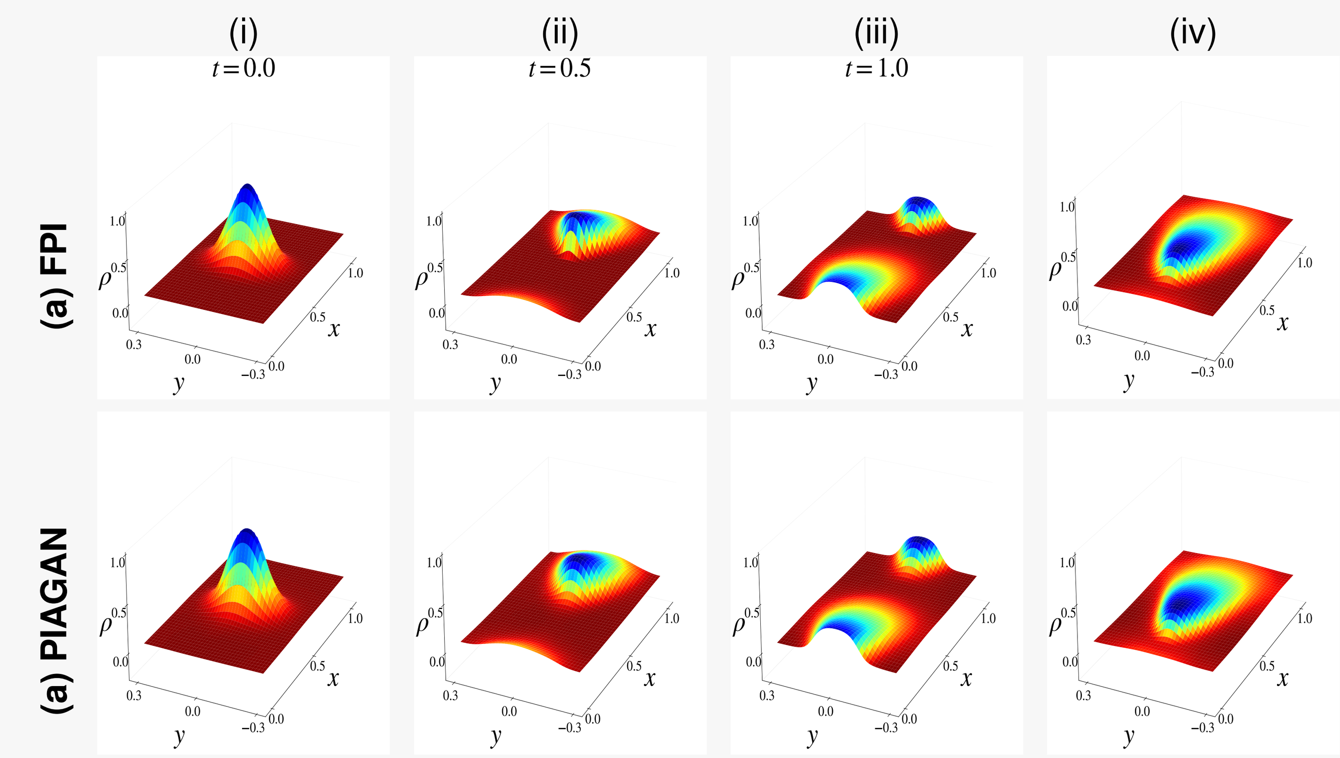


Architecture of physics-informed generative adversarial networks for MFG.

Experimental Results

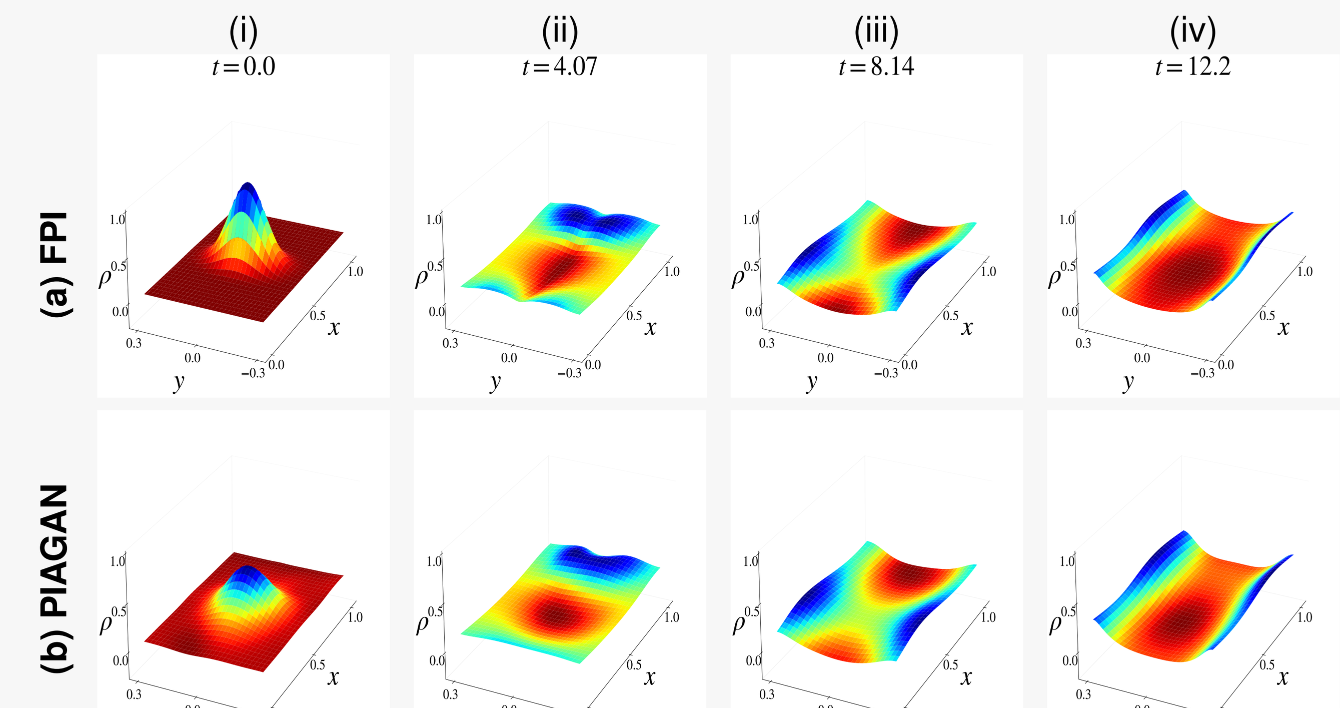
We utilise the proposed PIAGAN framework to model the EMFG and NE-MFG systems and compare the results with the fixed point iteration technique.

Equilibrium MFG



Evolution of traffic density obtained via (a) LWR model with Lax-Friedrichs Scheme, (b) LWR-MFG via FPI and (c) with PIAGAN model with parameters, $u_{max} = 1.02,\ v_{max} = 0.15,\ \rho_{max} = 1.13,\ \sigma(t) = 0.01,\ \kappa = 1.$

Non Equilibrium MFG: Quadratic Terminal Cost



Evolution of traffic density obtained for the robust NE-MFG system with (a) FPI and (b) PIAGAN method. The parameters of the robust NE-MFG are assumed to be $u_{max} = 1.02,\ v_{max} = 0.15,\ \rho_{max} = 1.13,\ \sigma(t) = 0.001,\ \kappa = 100$ and $f(y) = -\frac{y^2}{2}$.

References

[1] N. K. Pande, A. Kumar, and A. K. Gupta, "Robust mean field game of autonomous vehicles on multilane roadways via attention-based adversarial networks,"

[2] H. Cao, X. Guo, and M. Laurière, "Connecting gans, mean-field games, and optimal transport," *SIAM Journal on Applied Mathematics*, vol. 84, no. 4, pp. 1255–1287, 2024.

[3] M. Raissi, P. Perdikaris, and G. Karniadakis, "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations," *Journal of Computational Physics*, vol. 378, pp. 686–707, 2019.