Physics Informed Neural Network for Displacement Based Hyperelastic Problems

Ajay D. Borkar
Indian Institute of Technology Guwahati
a.borkar@alumni.iitg.ac.in

Dipjyoti Nath
Indian Institute of Technology Guwahati
dnath@iitg.ac.in

Sachin Singh Gautam
Indian Institute of Technology Guwahati
ssg@iitg.ac.in

Extended Abstract

In the present work, a physics-informed neural network (PINN) [1] is employed to solve hyperelastic problems that are subjected to displacement loads. A hyperelastic plate subjected to displacement at one end is considered as shown in Fig. 1. Deformation analysis of such a plate is done using the PINN model and the results are compared with finite element method (FEM).

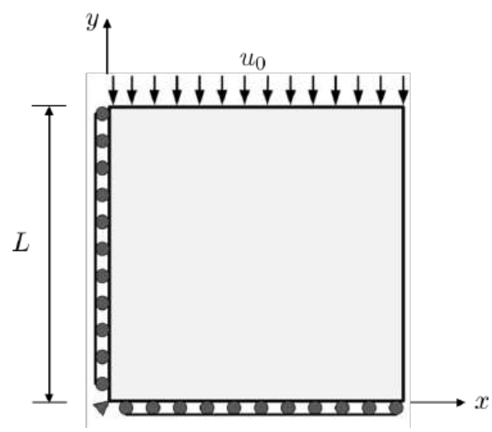

Figure 1: Configuration of hyperelastic plate (Neo-Hookean material) of length $L = 2$m under where initial displacement u_0 = -0.2 m is applied on the top face.

In the PINN model, the loss function is composed of the residuals from the governing equations as well as the boundary conditions. We use the deep collocation method (DCM) [2] to predict the displacement of the hyperelastic plate by minimizing the loss given as

$$\mathcal{L} = (\nabla_{\boldsymbol{X}} \boldsymbol{P} + \boldsymbol{f}_{\mathrm{B}})^2 + (\boldsymbol{u} - \boldsymbol{u}^*)^2 + (\boldsymbol{P} \cdot \boldsymbol{N} - \boldsymbol{t}^*)^2 \tag{1}$$

where $\boldsymbol{X}$: initial configuration, $\boldsymbol{P}$: first Piola-Kirchhoff stress, $\boldsymbol{f}_{\mathrm{B}}$: body force, $\boldsymbol{u}$: displacement, $\boldsymbol{u}^*$: prescribed displacement at boundary, $\boldsymbol{N}$: normal vector, and $\boldsymbol{t}^*$: applied traction. In Eq. 1, the first, second, and third terms refer to the residual of the governing equation, the Dirichlet boundary condition, and the Neumann boundary condition, respectively.

The network consists of 3 hidden layers with 30 hidden neurons in each layer. The network is trained initially for 1200 epochs using the Adam optimizer [3] and then trained for another 300 epochs with the L-BFGS optimizer [4] for better accuracy and faster convergence. This neural network predicts the deflections $(\boldsymbol{u}_x, \boldsymbol{u}_y)$ in x- and y-directions given the coordinates (x, y) in the domain. Figures 2

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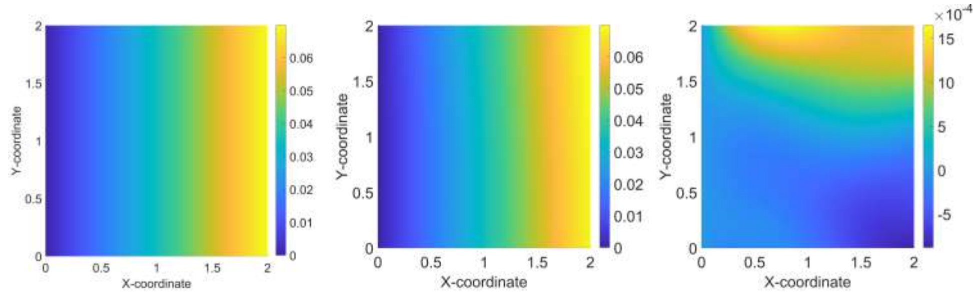
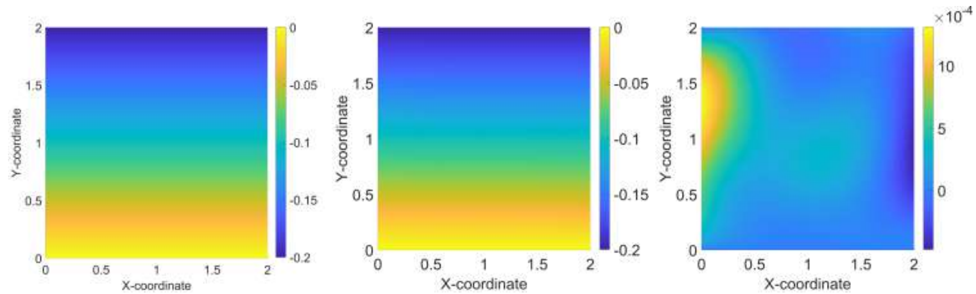
and 3 show the deflection contours of the plate obtained from the PINN model. No labeled data in terms of given deflections or stresses are used to train the PINN model.



(a) FEM solution ($\bar{\boldsymbol{u}}_x$) (b) PINN solution ($\boldsymbol{u}_x$) (c) Error $= \bar{\boldsymbol{u}}_x - \boldsymbol{u}_x$

Figure 2: Contour plots of x-deflection ($\boldsymbol{u}_x$) and the error of the plate over the domain.



(a) FEM solution ($\bar{\boldsymbol{u}}_y$) (b) PINN solution ($\boldsymbol{u}_y$) (c) Error $= \bar{\boldsymbol{u}}_x - \boldsymbol{u}_x$

Figure 3: Contour plots of y-deflection ($\boldsymbol{u}_y$) and the error of the plate over the domain.

The PINN model is able to capture the deformation of the hyperelastic plate. This study explores the use of PINN to analyze the nonlinear deformation of hyperelastic materials.

References

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